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of technique, and the behaviorist will be able to give a complete account of a subject's behavior both as regards *immediate response* to stimulation, which is effected through the larger muscles; *delayed response,* which is effected through the same muscles (so-called action after deliberation)—these two forms comprising what I have called *explicit behavior;* and the more elusive types, such as the movements of the larynx, which go on in cases where action upon stimulation is delayed (so-called thought processes). This latter form of behavior, which manifests itself chiefly in movements of the larynx, but which may go on in (to the eye) imperceptible form, in the fingers, hands, and body as a whole, I should call *implicit* behavior. For years to come, possibly always, we shall have to content ourselves with experimental observation and control of explicit behavior. I have a very decided conviction, though, that not many years will pass before implicit behavior will likewise yield to experimental treatment.

Possibly the most immediate result of the acceptance of the behaviorist's view will be the elimination of self-observation and of the introspective reports resulting from such a method.

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A NEW ALGEBRA OF IMPLICATIONS AND SOME CONSEQUENCES[1]

THE development of the algebra of logic has done more than emphasize the close relation of logic and mathematics. It has helped to show the possibility of an ideal development of pure mathematics in general, free—or nearly free—from tacit assumptions, parsimonious in its postulates, and absolutely rigorous in its methods of proof. In this ideal development, the algebra of implications, or "calculus of propositions," appears as the organon of proof in general, and hence as the necessary first step. The work of Russell and Whitehead and others has called attention to this method of procedure.[2] It is the logical outcome of the denial that mathematics must appeal to "construction" or any other empirical datum, once its postulates are laid down.

From this point of view, the drawing of conclusions is not a process in which premises retire into somebody's reasoning faculty and emerge in the form of the result; nor is the conclusion obtained through any subtle appeal to the perceptual character of space or of collections of marbles or arrays of fingers. Proof takes place through the collusion of two factors; first, postulates or propositions of the particular mathematical system in hand; secondly, postulates or theorems which state implication relations between premises of that logical or mathematical type and the desired conclusion. A mathematical operation is ideally no more than this: the substitution of the variables or functions of variables of the particular system—say, of cardinal number—for the logical variables in some proposition about implications. This proposition is more than a rule for inference; when the substitution is made, it *states* the implication. The result is the statement of what the variables or functions of the cardinal number system imply—a proposition in cardinal arithmetic. This result is not strictly the theorem to be proved, but only the statement that certain expressions or relations of variables imply certain others. Thus pure mathematics does not seek to prove theorems, but only, in the last analysis, that certain postulates imply certain theorems. And the proposition which states the particular implication relation—in more general form, because its variables have a wider range of meaning—is itself a mathematical proposition, in the algebra of logic.

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[1] Read in brief before the American Mathematical Society, San Francisco Section, October 26, 1912.

[2] "Principia Mathematica," Whitehead and Russell, intends to exhibit just this development of at least the fundamental branches of mathematics.

Pure mathematics is not concerned with the truth either of postulates or of theorems: so much is an old story. But just here a curious reservation seems necessary. Modern geometry—Euclidean or non-Euclidean—is not concerned with the truth either of postulates or of theorems, but it *is* concerned with the fact that the postulates *truly imply* the theorems. It would seem, then, that pure mathematics must concern itself with the *truth* of the *propositions in logic* which state, in general form, the implications in question. Since the logic, also, does not prove its theorems, but only proves that its postulates imply them—that they are true if the postulates are true—this concern is, finally, with the truth of postulates in the logic.

It may be objected that the logic, like any pure mathematics, is concerned only with "mathematical consistency." If so, it must be borne in mind that the algebra of implications ceases, in a sense, to be pure and becomes applied when its propositions are used in proving anything. Indeed, the attempt to separate formal consistency and material truth is, in the case of the logic, peculiarly difficult. For while other branches find their organon of proof in the logic, this discipline supplies its own. Hence, if this system is formally consistent, but contains a primitive proposition which is materially false, we shall have *false proofs* as well as materially false statements of implications, within the logic itself. Also a materially false theorem—either truly or falsely implied by the postulates—may, when used as a premise, lead to other false propositions; and when used as a *rule of inference,* to further false *proofs.* Thus a single materially false assumption in the logic might produce a numerous progeny early in life; and a "mathematically consistent" algebra of implications might still contain the basis for false proofs within itself, as well as in other branches.

Whoever has any interest in the rigorous development of pure mathematics may offer as warm a welcome to non-Euclidean geometry as to Euclidean, but can only view with alarm any algebra of logic whose postulates do not accord with the nature of valid inference.

Up to date, various sets of postulates for the algebra of logic have been offered, but all of the noteworthy ones produce mathematically equivalent systems, or systems which accord with the others so far as they go.[3] All of these which are complete contain theorems which have no application to our ordinary processes of reasoning and seem absurd to common sense.[4] The implication which figures in all such systems is what Mr. Russell has called "material implication," the difference of which from valid inference is undeniable.

The algebra to be proposed in this paper is applicable, in every proposition, to ordinary "sound reasoning," and it excludes all those theorems of material implication which are not so applicable. It has, consonantly, a different meaning of "implies," and proceeds from somewhat different assumptions. For the sake of brevity, this proposed system will be referred to as "strict implication."

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[3] Exception should be made of the system of Mr. H. MacColl, set forth in "Symbolic Logic and its Applications" (see also various articles in *Mind*).

Mrs. Ladd-Franklin's algebra makes inconsistency or exclusion the fundamental relation instead of implication. Nevertheless, any postulate or theorem of that system can be translated, *e. g.,* into a proposition of "Principia Mahtematica," and *vice versa,* provided the variables in both cases represent propositions. Mrs. Ladd-Franklin urges advantages of her algebra over those based on implication (see "Implication and Existence in Logic," *Philosophical Review,* Vol. XXI., pages 641–665), but these advantages are psychological and methodological rather than logical.

The system to be proposed in this paper has some points in common with the "formal implication" introduced by Mr. Russell in "Principles of Mathematics" and now included in "Principia Mathematica" (Vol. 1, page 132 and following), but it has also many points of difference. The most notable of these are: (1) Formal implication is based upon "material implication," and is immensely more complicated than the system to be proposed. (2) In formal implication, the negation of $(x) : \phi x . \supset . \psi x$ is equivalent to $(Ex) : \sim (\phi x . \supset . \psi x)$. In the proposed system $(x) : \phi x . \supset . \psi x$ is only a particular case of $p \supset q$ and its negative is simply $\sim (p \supset q)$. (3) As a consequence of (2) the proposed system does not treat, *e. g.,* "All men are mortals" as a case of implication, $(x) : \phi x . \supset . \psi x$, but as a relation of class inclusion.

[4] For a partial list of such theorems see "Interesting Theorems in Symbolic Logic," this JOURNAL, Vol. X., page 239. In that paper the number of such theorems is referred to as "apparently infinite." That it is actually infinite can be proved.

In order to make clear the different significance of strict implication from any system of material implication, it is necessary to introduce here some fairly tedious considerations. Material implication was preceded by, and historically grew out of, the "calculus of classes." But the algebra of classes contains an ambiguity which, comparatively harmless in itself, becomes serious when carried over into the algebra of implications.

In the algebra of classes, zero—in the usual notation—denotes the empty class or class with no members—the vertebrate invertebrates or the class of sea-serpents. But there are two kinds of such zero classes. Vertebrate invertebrates are impossible or *necessarily* non-existent: our fauna being what it is, the class of sea-serpents has no members. The class of Mondays that are Tuesdays is necessarily a zero class: the class of fair Mondays is a zero class in any month when it happens to rain every Monday.

Two further facts about the zero class are here worthy of attention. If $ac' = 0$ (if there are no a's which are not-c's), then all a's are c's. If ac' and $a'c$ are both zero classes, then a and c are equivalent or identical. They have the same extension, include the same members. If, now, all the Mondays are rainy and all the rainy days are Mondays, at some time, the class Mondays is identical with the class rainy days. But this relation is not a "necessary" one, since neither of the inclusions in this identity is a "necessary" inclusion. But consider a different identity. Let a be the class Mondays and c the days following Sundays. Then $ac' = 0$ and $a'c = 0$, $(ac' + a'c = 0)$, of necessity. All Mondays are days following Sundays and all days following Sundays are Mondays. We have here a necessary or definitive identity, an equivalence of *meaning* between "Monday" and "day following Sunday." The former identity is not of this type. If all Mondays are rainy days and all rainy days are Mondays, still that is no definition either of Monday or of rainy weather.

The defining relation (or pair of relations) is a relation of intension or meaning. To specify it requires the determination of two classes that are *necessarily* empty. This kind of class might be represented by equating it with a new symbol, -1. Such definitive or intensional equivalences are *not* specified when only extensional identity—the pair of $= 0$ relations—has been established.

The other point of importance is that the zero class is contained in every class. Since no Mondays are Tuesdays, the Mondays which are Tuesdays are also Fridays. If a is Mondays, b, Tuesdays, and c, Fridays, $(ab)c'=0$. It is also the case here that $ab=-1$. When it rains every Monday, all fair Mondays fall on the eighteenth. If a is Mondays, b, fair days, and c, the eighteenth, then $(ab)c'=0$, since $ab=0$. The questions here involved as to the legitimacy of inferring $(ab)c'=0$ from $ab=-1$, or from $ab=0$, we must not here pause to discuss. The important point is to note what the inclusion of the zero class in every class means when we pass to the algebra of implications.

The transition may be made as follows: The ordinary hypothetical proposition, "If *A* is *B*, *C* is *D*," may be taken to state either an inclusion of classes or an implication. If a represent the class of cases in which *A* is *B*, and c, the cases in which *C* is *D*, then the proposition states that class a is included in class c, or $ac'=0$. This same equation may also represent the fact that "*A* is *B*" implies "*C* is *D*." At once we have a new interpretation of the algebra in which it becomes the usual algebra of implications—material implication. The further details of this transition are familiar.

Let us now suppose that "*A* is *B*" is a false proposition. Then the class of cases in which *A* is *B* is (within the "universe of discourse") an empty class. $a=0$. If $a=0$, $ac'=0$, whatever proposition c may be. If a is false, a implies any proposition, c. If we suppose "*C* is *D*," c, to be true, its negative, c', is false. If $c'=0$, $ac'=0$, whatever proposition a may be. If c is true, any proposition, a, implies c. "Napoleon was an ancorite" implies "fresh vegetables are poison." The class of cases in which Napoleon was an ancorite is a zero class and is thus contained in the class of cases in which fresh vegetables are poison, or in any other class.

This is certainly not an implication relation of a sort which ought to figure in mathematical proof. Euclid's parallel postulate or Lobachevski's postulate about coplanars is—one or other of them—false. Nevertheless, he errs who would take either postulate to imply anything and everything. Logical consequences follow regardless of truth or falsity of premises.

Suppose we consider the meaning, for the algebra of implications, of $ac'=-1$. The cases of a which are not cases of c are *necessarily* an empty class; the case in which *A* is *B*, but *C* is not *D* is logically impossible. This means, if a, then necessarily c. The truth of "*A* is *B*" in conjunction with the falsity of "*C* is *D*" is impossible. This is exactly the situation when "*C* is *D*" can validly be inferred from "*A* is *B*." We may now add that if $a=0$ ("*A* is *B*" is false), then $ac'=0$: there is no case in which "*A* is *B*" is true, but "*C* is *D*" is false, since there is no case in which "*A* is *B*" is true. But $ac'=-1$ does not follow. If a (*A* is *B*) *were* true,

ac' might *not* be an empty class: the situation in which "*A* is *B*" would be true, but "*C* is *D*" false, may *not* be an *impossible* situation. "*C* is *D*" can not be inferred from "*A* is *B*" merely because "*A* is *B*" is false.

We have here, in $ac' = -1$, a new—or, rather, old and usual—meaning of "implies," and the basis of a new algebra of implications. The detailed criticism of material implication and its reconstruction as strict implication may be omitted in favor of results.

In order to avoid such expressions as $ac = 0$ and $ac = -1$, it will prove convenient to give them equivalents. $ac = 0$ means "It is false that a and c are both true"; hence also, "At least one of the propositions, a and c, is, in fact, false." This last expression is usually symbolized as $a' + c' = 1$, or simply $a' + c'$. $ac' = -1$ means "It is impossible that a and c be true together"; hence also, "At least one of the propositions, a and c, is, of necessity, false. This relation may be symbolized as $a' \vee c'$. The distinction between $a' + c'$ and $a' \vee c'$, or between $a + c$ and $a \vee c$, is this: $a \vee c$ represents a dilemma whose horns are a and c; $a + c$ has a wider meaning and covers such expressions as "Either Napoleon was not an ancorite or fresh vegetables are poison." This last is no "necessary" relation. It breaks down before the test of contrary to fact supposition.[5]

We may now exhibit the primitive ideas and postulates for the proposed system of Strict Implication.

PRIMITIVE IDEAS

1. *Propositions or propositional functions*, symbolized by p, q, r, etc.

A propositional function is an expression, containing a variable or variables, which becomes a proposition when a value of the variable or variables is assigned. It is one of the advantages of Strict Implication that propositions and propositional functions obey the same laws. We reason in the same "modes" about "x is a man" and "Socrates is a man."

2. *Negation.* $\sim p$ symbolizes the negation of p or "p is false."

3. *Implication.* $p \supset q$ symbolizes "q can be inferred from p" or "p (strictly) implies q."

4. *Product.* pq symbolizes the logical product of p and q, or the proposition which asserts both p and q; "p is true and q is true."

5. *Logical Constants.* Negation is a logical constant, and any relation which obtains between propositions or propositional func-

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[5] For a discussion of the ambiguity of "either—or" propositions in relation to the algebra, see "Implication and the Algebra of Logic," C. I. Lewis, *Mind*, N. S., No. 84.

tions as such is a logical constant. Implication and product are such; other relations will be defined.

POSTULATES

P. 1. $(p = q) \supset [(p \supset q)(q \supset p)]$.
P. 2. $[(p \supset q)(q \supset p)] \supset (p = q)$.
P. 3. $(p \vee q) = ({\sim} p \supset q)$. Definition of the dilemmatic disjunction, $p \vee q$, —$pq' = -1$.
P. 4. $(p + q) = {\sim} ({\sim} p {\sim} q)$. Definition of the non-dilemmatic disjunction, $p + q$, —$pq' = 0$.
P. 5. $(p + p) \supset p$.
P. 6. $(p \vee q) \supset (q \vee p)$.
P. 7. $qp \supset pq$.
P. 8. $pq \supset p$.
P. 9. $p \supset {\sim} ({\sim} p)$.
P. 10. $(p \vee q) \supset (p + q)$. The dilemmatic disjunction of two propositions implies their non-dilemmatic disjunction as well. This implication is not reversible.
P. 11. $[p \vee (q \vee r)] \supset [q \vee (p \vee r)]$.
P. 12. $(q \supset r) \supset [(p \supset q) \supset (p \supset r)]$.

(Adjoined remarks in the above are, of course, no part of the postulates.)

P. 13. If p is asserted and $p \supset q$ is asserted, q may be asserted.
P. 14. If p and q are separately asserted, pq may be asserted.
P. 15. Any expression which contains only propositions, or propositional functions, and logical constants is itself a proposition, or propositional function.
P. 16. Any proposition or propositional function may be substituted for p or q or r, etc., in any postulate or theorem.

The last four postulates are the principles of operation of the system. The first two practically restate what is already contained in the primitive ideas of implication and product. The last two determine the range of the variables p, q, r, etc.

Optional postulates which might be added to our list are:

P. 17. $[p(qr)] \supset [q(pr)]$.
P. 18. $(q \supset r) \supset [(p + q) \supset (p + r)]$.
P. 19. $[p \vee (q + r)] \supset [q \vee (p + r)]$.

Of these, the first two have only infrequent and unimportant consequences. P. 19 has highly important consequences. The truth of every one of these turns upon the question as to what would be true in case logical principles were contradicted by facts. The truth of the postulate is doubtful and its significance chiefly epistemological.

It may be remarked in passing that P. 19 and its consequences well exemplify the fact that a formal postulate may sometimes serve to conceal rather than to express an important principle.

The entire system of strict implication could be developed from a somewhat smaller number of postulates, by a different choice of primitive ideas. But the set given above has compensating advantages.

Among the immediate consequences of these postulates, the following are selected as those which offer some difficulty of deduction but are important for further work. The order in which they are set down indicates, in general, the order of their simplest proof.

P. 20. $(p = q) \supset (p \supset q)$.
P. 21. $(p = q) \supset (q \supset p)$.
P. 22. $(\sim p \supset q) \supset (\sim q \supset p)$.
P. 23. $p \supset (p + q)$.
P. 24. $p \supset p$.
P. 25. $\sim(\sim p) \supset p$.
P. 26. $(p \supset q) \supset (\sim p \vee q)$.
P. 27. $(p + q) \supset (q + p)$.
P. 28. $(p \vee p) \supset p$.
P. 29. $(\sim p \supset p) \supset p$.
P. 30. $(p \supset q) \supset \sim (p \sim q)$.

P. 28, P. 29, and P. 30 state implications which are *not,* in strict implication, reversible; in any system of material implication they would be reversible. This difference is of considerable importance. The general character of further consequences of these postulates is indicated by the fact that strict implication includes all theorems of material implication which are consonant with the meaning of "implies" which we have chosen, and many other significant theorems which do not appear in material implication. Strict implication excludes all theorems of material implication which are false for ordinary inference. Besides excluding such doubtful theorems, strict implication makes clear the meaning of many others which seem doubtful because of the ambiguity of $p + q$, in systems of material implication, if read "either p or q." To take only two examples: (1) $\sim(p + q) \supset (\sim p + q)$;[6] (2) $\sim(p + q) \supset (p + \sim q)$:[7] both these theorems become clear when we discover that the disjunctions involved will not be dilemmas,—that the theorems do not hold in the form (1) $\sim(p \vee q) \supset (\sim p \vee q)$ and (2) $\sim(p \vee q) \supset (p \vee \sim q)$.

Strict implication has, also, positive results which are equally important. These may be brought out by considering P. 1 and P. 2. Objection might be taken to these postulates on the ground that they contain an undefined expression $(p = q)$ which is not mentioned among the primitive ideas. P. 3 and P. 4 are expressions of this type, and all such might be referred to as "definitions." The curious fact is that it is not essential, in strict implication, that definitions should have any meaning; it is requisite only that we should be able to get their implications. And these may be had, for any definition, by applying P. 1. If we substitute, in P. 1 $(p \vee q)$, for p and $(\sim p \supset q)$ for q, we shall have,

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[6] "Principia Mathematica," *2·47.

[7] *Ibid.,* * 2·48.

$$[(p \vee q) = (\sim p \supset q)] \\ \supset \{[(p \vee q) \supset (\sim p \supset q)][(\sim p \supset q) \supset (p \vee q)]\}.$$

The first part of this expression is, now, the "definition," P. 3. The second part, which P. 3 is here asserted to imply, is a pair of implications. By using P. 7 and P. 8 upon P. 1, it could be shown how we may get either of these implications separately (see P. 20 and P. 21). That is, the "definition," P. 3, merely as an undefined expression containing functions of p and q, implies these two theorems, (1) $(p \vee q) \supset (\sim p \supset q)$, (2) $(\sim p \supset q) \supset (p \vee q)$. The only use to which any "definition" $(p = q)$ may be put, is to get from p to q or from q to p. This is properly done by using one of the implications $(p \supset q)$ and $(q \supset p)$. Thus "definitions" could be dispensed with altogether, and replaced by the pairs of implications which we get from them by P. 1. If this procedure were adopted, P. 1 and P. 2 would be superfluous.

The use made of definitions in any system of material implication, and in mathematics generally, is to substitute the defining expression for the defined, or *vice versa,* at any point in the "work." This procedure has been defended on the ground that the two expressions represent the same mathematical entity. But it is extremely doubtful if the only reason for any definition is that we may have various symbolic representations of the same thing. Definitions would then seem a useless violation of the principle of parsimony. Especially this is true where the defining expression is not symbolically simpler than the defined. In strict implication, and in any mathematical system which used it as an organon of proof, such substitution of expressions equivalent by definition is wholly unnecessary. Instead, one of the pairs of implications involved is made use of, and is treated exactly like any other implication. This brings one step nearer the ideal of mathematical work as consisting only in the substitution for variables of values of variables, or of other variables which may be regarded as values.

An alternative treatment of definitions would be to add to the number of our primitive ideas, the relation "is logically equivalent to" or "means the same as," symbolized by $=$. This procedure avoids various possible objections to "expressions without meaning" and yet retains the advantage that substitution of equivalent expressions is unnecessary. Adopting this method, we can then make a somewhat spectacular deduction from P. 1 and P. 2. If we substitute in P. 2, the product qr, for q, we have (theorem 1):

$$[(p \supset qr)(qr \supset p)] \supset (p = qr).$$

Substituting, in this theorem, $(p = q)$ for p, $(p \supset q)$ for q, and $(q \supset p)$ for r, we have:

$$[\{(p = q) \supset [(p \supset q)(q \supset p)]\}\{[(p \supset q)(q \supset p)] \supset (p = q)\}] \\ \supset \{(p = q) = [(p \supset q)(q \supset p)]\}.$$

The expression within the first pair of braces is now P. 1; within the second pair, P. 2. Hence this complicated expression may be read: Theorem 1 states that the product, (P. 1)(P. 2), implies as a theorem:

$$(p = q) = [(p \supset q)(q \supset p)].$$

This theorem may be read: "p means the same as q" means the same as "p implies q and q implies p." (The use of postulates P. 13–P. 16 in this proof is tacit.)

It would be unsafe to call this theorem a definition of the defining relation. Let us be satisfied to say that we have deduced an expression which makes clearer the meaning of that logical equivalence which was taken as a primitive indefinable.

This theorem is an interesting puzzle, but is unnecessary to strict implication, since the only use which would be made of it would be to derive its implications, which are P. 1 and P. 2. Other equivalences can, in strict implication, be deduced. For example, if we substitute q for p and p for q in P. 6, we have $(q \vee p) \supset (p \vee q)$. Without substitution, P. 6 reads $(p \vee q) \supset q \vee p)$. These two together imply, by P. 2, $(p \vee q) = (q \vee p)$. A dilemma means the same thing when the order of its members is reversed. This theorem is definitive of the dilemma just so far as the expressions, $(p \vee q)$ and $(q \vee p)$, are different. The theorem $(p \supset q) = (\sim p \vee q)$ can also be deduced. If we can not derive definitions, at least we derive equivalences of meaning, of which definitions are cases. Is it not possible that the usual treatment of definitions involves some confusion between mathematical and psychological requirements?

If what is set forth in the above paragraphs be correct, it throws considerable light on the nature of definitions in mathematics. Any definition is an equivalence of meaning between two expressions,—a reciprocal implication when "implies" has the meaning which figures in valid inference. "$p = q$ Def." means $(pq' = -1)$ and

$(p'q = -1)$. Definitions are not assumptions fundamentally different in kind from other postulates. They need not stand outside the mathematical system as sanction for substitutions, but should appear *in* the system and be used exactly as other postulates. Such equivalences can frequently be deduced, though the psychological requirement, that expressions must be clear in meaning when first introduced, limits the usefulness of such deductions. Definitions could be dispensed with, without materially altering the system, if proper implications were assumed.

It should be remarked, however, that strict implication does not stand or fall with this treatment of definition which it makes possible. The more usual method might have been followed in developing the system.

In leaving the subject of equivalences, it is well to note the difference between $(p = q)$ in strict implication and the "equivalence" ("Principia Mathematica," $p \equiv q$) which figures in material implication. This last is not an equivalence of meaning, but only of "truth value." $(p \equiv q)$ means only, "p and q are either both true or both false." This relation holds for $(p = q)$, but the defining relation can not be got out of it. For this reason, "equal by definition" is, in material implication, a primitive idea which needs to be distinguished from equivalence in general. The fact that equivalence of *truth value* is reciprocal *material* implication, while equivalence of *meaning* is reciprocal *strict* implication, throws more light upon the meaning of "implies" in the different systems. Which of these accords with the nature of inference, is a question of some importance for pure mathematics, if any ideal development from "logical constants" is to be attempted.

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DISCUSSION

"EVERYBODY'S WORLD" AND THE WILL TO BELIEVE

I AM not sure that these few lines, elicited by Mr. Adams's courteous paper,[1] should come under the head of "discussion." They are not intended as a refutation of what he has said, but, rather, as an explanation of one or two points on which I do not seem to have made my meaning clear.

But, first, I should confess that I regret having headed a chapter in my recent book with the words: "The World of the New Realism,"

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[1] This JOURNAL, Vol. X., page 186.